MATHEMATICS

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AIM POINTS
MATHEMATICS
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XIth, XIIth, TARGET IIT-JEE (MAIN + ADVANCE) & COMPETITIVE EXAM. FOR XII (PQRS)

ALGEBRA OF VECTORS
& Their Properties

CONTENTS

THINGS TO REMEMBER

1. A vector is a physical quantity having both magnitude and direction.

2. If $\vec{a}, \vec{b}, \vec{c}$ are the vectors represented by the sides of a triangle taken in order, then $\vec{a}+\vec{b}+\vec{c}=\vec{0}$.

   Conversely, if $\vec{a}, \vec{b}, \vec{c}$ are three non-collinear vectors, such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then they form thesides of a triangle taken in order.

3. Two non-zero vectors $\vec{a}$ and $\vec{b}$ are collinear iff there exist non-zero scalars x and y such that $x\vec{a}$ + $y\vec{b} = \vec{0}$.

4. If $\vec{a}$ and $\vec{b}$ are two non-zero non-collinear vectors, then

   $x\vec{a} + y\vec{b} = \vec{0} \quad \Rightarrow x = y = 0$

5. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors, thenany vector $\vec{r}$ coplanar with $\vec{a}$ and $\vec{b}$ can be uniquely expressed as

   $\vec{r} = x\vec{a} + y\vec{b}$, where x, y are scalars

   or, $\vec{r} = \{x\,|\vec{a}|\}\ \vec{a} + \{y\,|\vec{b}|\}\vec{b}$

6. If $\vec{a}, \vec{b}, \vec{c}$ are three given non-coplanar vectors, then every vector $\vec{r}$. in space can be uniquely expressed as

   $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$ for some scalars x, y and z.

   or, $\vec{r} = \{x\,|\vec{a}|\}\ \vec{a} + \{y\,|\vec{b}|\}\vec{b} + \{z|\vec{c}|\}\vec{c}$

   Here, vectors $x\vec{a}$,$y\vec{b}$ and $z\vec{c}$ are called the components of $\vec{r}$ in the directions of $\vec{a}, \vec{b}$ and $\vec{c}$ respectively and the scalars x, y and z are knows as the coordinates of $\vec{r}$ relative to the triad of non-coplanar vectors $\vec{a}, \vec{b}, \vec{c}$. The triad of non-coplanar vectors $\vec{a}, \vec{b}, \vec{c}$ relative to which we de-compose any vector $\vec{r}$ is called a base.

   The scalars $x\,|\vec{a}|$, $y\,|\vec{b}|$, $z\,|\vec{c}|$ are known as the projections of $\vec{r}$ in the directions of $\vec{a}, \vec{b}, \vec{c}$ respectively.

7. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero non-coplanar vectors and x, y, z are three scalars, then $x\vec{a} + y\vec{b} + y\vec{c} = \vec{0}$ $\Rightarrow x = y = z = 0$.

8. If *l, m, n* are direction cosines of a vector $\vec{r}\,(=\overline{OP})$, where O is the origin and the point P has (x, y, z) as its coordinates, then

   (i) $l^2 + m^2 + n^2 = 1$

   (ii) $x = l\,|\vec{r}|,\ y = m|\vec{r}|,\ z = n\,|\vec{r}|$

   (iii) $\vec{r} = |\vec{r}|\,(l\hat{i} + m\hat{j} + n\hat{k})$

   (iv) $\vec{r} = l\hat{i} + m\hat{j} + n\hat{k}$

9. If $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$, then a, b, c are proportional to its direction ratios and its direction cosines are $\frac{\pm a}{\sqrt{a^2+b^2+c^2}}, \frac{\pm b}{\sqrt{a^2+b^2+c^2}}, \frac{\pm c}{\sqrt{a^2+b^2+c^2}}$

10. (i) A set of non-zero vectors $\vec{a_1}, \vec{a_2}, \vec{a_3}, ..., \vec{a_n}$ is linearly independent, if $x_1\vec{a_1} + x_2\vec{a_2} + ... + x_n\vec{a_n} = \vec{0}$ $\Rightarrow x_1 = x_2 = ... = x_n = 0$

    (ii) A set of vectors $\vec{a_1}, \vec{a_2}, \vec{a_3}, ..., \vec{a_n}$ is linearly dependent, if three exist scalars $x_1\vec{a_1} + x_2\vec{a_2} + ... + x_n\vec{a_n} = \vec{0}$

    (iii) Any two non-zero, non-collinear vectors are linearly independent.

    (iv) Any two collinear vectors are linearly dependent.

    (v) Any three non-coplanar vectors are linearly independent.

    (vi) Anythree coplanar vectors are linearly dependent.

    (vii) any set of four or more vectors in three dimensional space is linearly dependent set.

11. If A and B are points with position vectors $\vec{a}$ and $\vec{b}$ respectively, then the position vector of a point C dividing AB in the ratio m : n

    (i) internally, is $\frac{m\vec{b} + n\vec{a}}{m+n}$

    (ii) externally, is $\frac{m\vec{b} - n\vec{a}}{m-n}$

12. If A and B are two points with position vectors $\vec{a}$ and $\vec{b}$ respectively and m, n are positive real numbers, then

    $m\overrightarrow{OA} + n\overrightarrow{OB} = (m+n)\overrightarrow{OC}$,

    where C is a point on AB dividing it in the ratio n. m.

    Also, $\overrightarrow{OA} + \overrightarrow{OB} + 2\overrightarrow{OC}$, where C is the mid-point of AB.

13. If S is any point in the plane of a triangle ABC, then

    $\overrightarrow{SA} + \overrightarrow{SB} + \overrightarrow{SC} = 3\overrightarrow{SG}$,

    where G is the centroid of ΔABC.

14. The necessary and sufficient condition for three points with position vectors $\vec{a}, \vec{b}, \vec{c}$ to be collinear is that three exist scalars x, y, z not all zero such that

    $x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$, where $x + y + z = 0$

15. The necessary and sufficient condition for four point with position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ to be coplanar is that there exist scalars, x, y, z, t not all zero such that

    $x\vec{a} + y\vec{b} + z\vec{c} + t\vec{d} = \vec{0}$, where $x + y + z + t = 0$

EXERCISE-1

1. Classify the following measures as scalars and vectors :
   (i) 15 kg (ii) 20 kg weight (iii) 45°
2. Classify the following as scalars and vector quantities :
   (i) time period (ii) distance (iii) displacement (iv) force
   (v) Work (vi) Velocity (vii) Acceleration
3. In this section, we shall learn some properties of addition of vectors.
   Commutatativity : For any two vectors $\vec{a}$ and $\vec{b}$, we have $\vec{a}+\vec{b}=\vec{b}+\vec{a}$
4. If $\vec{a}, \vec{b}, \vec{c}$ be the vectors represented by the sides of a triangle, taken in order, then prove that $\vec{a}+\vec{b}+\vec{c}$.
5. If P_1, P_2, P_3, P_4 are points in a plane or space and O is the origin of vectors, show that P_4 coincides with O iff $\overrightarrow{OP_1}+\overrightarrow{P_1P_2}+\overrightarrow{P_2P_3}+\overrightarrow{P_3P_4}=\vec{0}$.
6. If $\overrightarrow{OP}+\overrightarrow{OQ}=\overrightarrow{QO}+\overrightarrow{OR}$, show that the points P, Q, R are collinear.
7. If $\vec{a}, \vec{b}$ are any two vectors, then give the geometrical intepretation of the relation $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$
8. If $\vec{a}$ and $\vec{b}$ are the vectors determined by two adjacent side of a regular hexagon, what are the vectors determined by the other taken in order ?
9. If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.
10. If $\vec{a}$ and $\vec{b}$ represent two adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ respectively of a paralleogram ABCD, them show that is diagonals $\overrightarrow{AC}$ and $\overrightarrow{DB}$ are equal to $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ respectively.
11. Vectors drawn from the origin O to the points A, B and C are respectively $\vec{a}, \vec{b}$ and $4\vec{a}-3\vec{b}$. Find $\overrightarrow{AC}$ and $\overrightarrow{BC}$.
12. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are distinct non-zero vectors represented by directed line segments from the origin to the points A, B, C and D respectively, and if $\vec{b}-\vec{a}=\vec{c}-\vec{d}$, then prove that ABCD is a parailelogram.
13. A, B, P, Q and R are five points in a plane. Show that the sum of the vectors $\overrightarrow{AP}$, $\overrightarrow{AQ}$, $\overrightarrow{AR}$, $\overrightarrow{PB}$, $\overrightarrow{QB}$ and $\overrightarrow{RB}$ is 3 $\overrightarrow{AB}$.
14. Let A and B be two points with position vectors $\vec{a}$ and $\vec{b}$ respectively, and let C be a point dividing AB internally in the ration m : n. Then the position vector of C is given by

$$\overrightarrow{OC}=\frac{m\vec{b}+n\vec{a}}{m+n}$$

15. Find the position vector of a point R which divides the line segment joining P and Q whose position vectors are $2\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$, externally in the ratio 1 : 2. Also, show that P is the mid-point of the line segment R.Q.

16. Four points A, B, C, D with position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively are such that $3\vec{a} - \vec{b} + 2\vec{c} - 4\vec{d} = 0$. Show that the four points are oplanar. Also find the position vector of the point of intersection of lines AC and BD.

17. Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of three distinct points A, B, C. If there exist scalar x, y, z (not all zero) such that $x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$ and $x + y + z = 0$, then show that A, B and C lie on a line.

18. Points L, M, N divide the side BC, CA, AB of ΔABC in the ratio 1:4, 3:2, 3:7 respectively. Prove that $\overrightarrow{AL} + \overrightarrow{BM} + \overrightarrow{CN}$ is a vector parallel to $\overrightarrow{CK}$, where K divides AB in the ratio 1:3.

19. Prove using vectors : Medians of a triangle are concur rent.

20. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \vec{0}$.

21. If D and E are the mid-points of sides AB and AC of a triangle ABC respectively, show that $\overrightarrow{BE} + \overrightarrow{DC} = \frac{3}{2}\overrightarrow{BC}$.

22. If ABC and A'B'C' are two triangles G, G' be their centroids, prove that $\overrightarrow{AA'} + \overrightarrow{BB'} + \overrightarrow{CC'} = 3\overrightarrow{GG'}$.

23. Prove that the sum of the vectors directed from the vertices to the mid-points of oppositive sides of a triangle is zero.

24. Show that the line joining one vertex of a parallelogram to the mid-point of an opposite side trisects the diagonal and is trisected thereat.

25. Prove using vectors : The diagonals of a quadrilateral bisect each other iff it is a parallelogram.

26. Using vector method, prove that the line segments joining the mid-points of the adjacent sides of a quadrilateral taken in order from a parallelogram.

27. Prove by vector method that the line segment joining the mid-points of the dingonals of a trapezium is parallel to the parallel sides and equal to half of their difference.

28. If ABCD is quadrilateral and E and F are the mid-points of AC and BD respectively prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$

29. ABCD is a parallelogram. E, F are mid-points of BC, CD respectively. AE, AF mear the diagonal BD at points Q and P respectively. Show that points P and Q trisect DB.

30. ABCD is a parallelogram. If L and M are the mid-points of BC and DC respectively, them express $\overrightarrow{AL}$ and $\overrightarrow{AM}$ in terms of $\overrightarrow{AB}$ and $\overrightarrow{AD}$. Also, prove that $\overrightarrow{AL} + \overrightarrow{AM} = \frac{3}{2}\overrightarrow{AC}$.

31. If P and Q are the mid-points of the sides AB and CD of a parallelogram ABCD, prove that DP and BQ cut the diagonal AC in its points of trisection which are also the points of trisection of DP and BQ respectively.

32. If O is the circumcentre and O' the orthocentre of a triangle ABC, prove that

    (i) $\overrightarrow{SA} + \overrightarrow{SB} + \overrightarrow{SC} = 3\overrightarrow{SG}$, where S is any point in the plane of triangle ABC whose centroid is at G.

    (ii) $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + = \overrightarrow{OO'}$

(iii) $\overrightarrow{O'A}+\overrightarrow{O'B}+\overrightarrow{O'C}+=2\overrightarrow{O'O}$

(iv) $\overrightarrow{AO'}+\overrightarrow{O'B}+\overrightarrow{O'C}+=\overrightarrow{AP}$, where $\overrightarrow{AP}$ is the diameter of the circumcircle.

33. ABCD is a parallelogram and P is is the point of intersection of its diagonals. If O is the origin of reference, show that

$\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+OD=4\overrightarrow{OP}$

34. Show that the four points P, Q, R, S with position vector $\vec{p},\vec{q},\vec{r},\vec{s}$ respectively such that $\overrightarrow{5p}-\overrightarrow{2q}+\overrightarrow{6r}-\overrightarrow{9s}=\vec{0}$, are coplanar. Also, find the position vector of the point of intersection of the lines PR and QS.

35. Find the values of x and y so that the vectors $2\hat{i}+3\hat{j}$ and $x\hat{i}+y\hat{j}$ are equal.

36. Let O be the origin and let P(–4, 3) be a point in the xy-plane. Express $\overrightarrow{OP}$ in terms of vectors $\hat{i}$ and $\hat{j}$. Also, find $|\overrightarrow{OP}|$.

37. Find the scalar and vector components of the vector with initial point A (2, 1) and lerminal point B(–5, 7).

38. If the position vector $\vec{a}$ of the point (5, n) is such that $|\vec{a}|$ = 13, find the value of n.

39. Using, vectors, show that the points A(–2, 1), B(–5, –1) and C(1, 3) are collinear.

40. If $\vec{a}$ is a position vector whose tip is (–1, –3). Find the coordinates of the point B such that $\overrightarrow{AB}=\vec{a}$. If A has coordinates (–1, –5).

41. Find a vector of magnitude 5 units which is parallel to the vector $2\hat{i}-\hat{j}$.

42. Write all the unit vectors in XY-plane.

43. Write down a unit vector in XY-plane, making an angle of 30° with the position direction of x-axis.

44. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of northand stops. Determine the girl's displacement from her initial point of departure.

45. If the position vectors of the points A(3, 4), B(5, –6) and C(4, –1) are $\vec{a},\vec{b},\vec{c}$ respectively, compute $\vec{a}+2\vec{b}-3\vec{c}$.

46. Show that the points $2\hat{i}$, $-\hat{i}-4\hat{j}$ and $-\hat{i}+\sqrt{3}\,\hat{j}$.

47. Find the value of x, y and z so that the vectors $\vec{a}=x\hat{i}+2\hat{j}+z\hat{k}$ and $\vec{b}=2\hat{i}+y\hat{j}+\hat{k}$ are equal.

48. Find the magnitude of the vector $\vec{a}=3\hat{i}-2\hat{j}+6\hat{k}$.

49. Find the unit vector in the direction of $\vec{a}+\vec{b}$, if $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}-\hat{k}$.

50. Find the unit vector in the direction of $\overrightarrow{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6) respectively.

51. Show that the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $-4\hat{i}+6\hat{j}-8\hat{k}$ are collinear.

52. The position vectors of the points P, Q, R are $\hat{i}+2\hat{j}+3\hat{k}$, $-2\hat{i}+3\hat{j}+5\hat{k}$ and $7\hat{i}-\hat{k}$ respectively. Prove that P, Q and R are collinear.

53. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ represent two adjacent sides of a paral lelogrant, find unit vectors parallel tothe diagonals of the parallelogram.

54. Show that the points A(6, –7), B(16, –19, –4), C(0, 3, –6) and D(2, –5, 10) are such that AB and CD intersect at the point P(1, –1, 2).

55. Show that the points A, B and C with position vectors $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$ respectively, form the vertices of a right angled triangle.

56. The adjacent sides of a parallelogram are represented by the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=-2\hat{i}+\hat{j}+2\hat{k}$. Find unit vectors parallel to the diagonals of the parallelogram.

57. Find the position vector of a point R which divides the line segment joining points $P(\hat{i}+2\hat{j}+\hat{k})$ and $Q(-\hat{i}+\hat{j}+\hat{k})$ in the ratio 2 : 1.
    (i) internally
    (ii) externally

58. Show that the points $A(2\hat{i}-\hat{j}+\hat{k})$, $B(\hat{i}-3\hat{j}-5\hat{k})$, $C(3\hat{i}-4\hat{j}-4\hat{k})$ are the vertices of a right angled triangle.

59. Find the position vector of the mid-point of the vector joining the points P(2, 3, 4) and Q(4, 1, –2).

60. Find the value of x for which $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector.

61. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$, find a unit vector parallel to $2\vec{a}-\vec{b}+3\vec{c}$.

62. Show that the points A(1, –2, –8), B(5, 0, –2) and C(11, 3, 7) are collinear, and find the ratio in which B divides AC.

63. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.

64. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, find the value of x for which the vectors $\vec{\alpha}=(2x+1)\vec{a}-\vec{b}$ and $\vec{\beta}=(x-2)\vec{a}+\vec{b}$ are collinear.

65. If $\vec{a},\vec{b},\vec{c}$ are three non-null vectors such that any two of them are non-collinear. If $\vec{a}+\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+\vec{c}$ is collinear with $\vec{a}$, then find $\vec{a}+\vec{b}+\vec{c}$.

66. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors such that any two of them are non-collinear. If $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$, then prove that $\vec{a}+2\vec{b}+6\vec{c}=0$.

67. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors and vectors $\vec{\alpha}=(x+4y)\vec{a}+(2x+y+1)\vec{b}$ and $\vec{\beta}=(-2x+y+2)\vec{a}+(2x-3y-1)\vec{b}$ are connected by the relation $3\vec{\alpha}=2\vec{\beta}$, find x, y.

68. Let Let $\vec{u}=\hat{i}+2\hat{j}, \vec{v}=2\hat{i}+\hat{j}$ and $\vec{w}=4\hat{i}+3\hat{j}$, . Find scalars x and y such that $\vec{w}=x\vec{u}+y\vec{v}$.
69. Show that the points with position vectors $\vec{a}-2\vec{b}+3\vec{c}$, $-2\vec{a}+3\vec{b}-\vec{c}$ and $4\vec{a}+7\vec{b}-7\vec{c}$ are collinear.
70. If the points with position vectors $60\hat{i}+3\hat{j}$, $40\hat{i}+8\hat{j}$ and $a\hat{i}+11\hat{j}$ are collinear, find the value of a.
71. If the points with position vectors $10\hat{i}+3\hat{j}$, $12\hat{i}+5\hat{j}$ and $a\hat{i}+11\hat{j}$ are collinear, find the value of a.
72. If $\vec{AO}+\vec{OB}+\vec{OC}$, prove that A, B, C are collinear points.
73. $\vec{a},\vec{b},\vec{c}$ are three non-zero non-coplanar vectors and x, y, z are three scalars, then
74. Show that the vectors $2\vec{a}-\vec{b}+3\vec{c}$, $\vec{a}+\vec{b}-2\vec{c}$ and $\vec{a}+\vec{b}-3\vec{c}$ are non-coplanar vectors.
75. Show that the points whose position vectors are given, are collinear :

    (i) $2\hat{i}+\hat{j}-\hat{k}$, $3\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}+4\hat{j}-3\hat{k}$

    (ii) $3\hat{i}-2\hat{j}+4\hat{k}$, $\hat{i}+\hat{j}+\hat{k}$ and $-\hat{i}+4\hat{j}-2\hat{k}$
76. Using vector method, prove that the following points are collinear :
    (i) A(6, –7, –1), B(2, –3, 1) and C(4, –5, 0)
    (ii) A(2, –1, 3), B(4, 3, 1) and C(3, 10, –1)
    (iii) A(1, 2, 7), B(2, 0, 3) and C(3, 10, –1)
    (iv) A(–3, –2, –5), B(1, 2, 8) and C(3, 4, 7)
    (v) A(2, –1, 3), B(3, –5, 1) and C(–1, 11, 9)
77. If $\vec{a},\vec{b},\vec{c}$ are non-zero, non-coplanar vectors, prove that the following vectors are coplanar :

    (i) $5\vec{a}+6\vec{b}+7\vec{c}$, $7\vec{a}-8\vec{b}+9\vec{c}$ and $3\vec{a}+20\vec{b}+5\vec{c}$

    (ii) $\vec{a}-2\vec{b}+3\vec{c}$, $-3\vec{b}+5\vec{c}$ and $-2\vec{a}+3\vec{b}-4\vec{c}$
78. Show that the four points having position vectors $6\hat{i}-7\hat{j}$, $16\hat{i}-19\hat{j}-4\hat{k}$, $3\hat{j}-6\hat{k}$, $2\hat{i}-5\hat{j}+10\hat{k}$ are coplanar.
79. Prove that the following vectors are complanar

    (i) $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$

    (ii) $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+3\hat{j}-\hat{k}$ and $-\hat{i}-2\hat{j}+2\hat{k}$
80. Prove that the following vectors are non-coplanar :

    (i) $3\hat{i}+\hat{j}-\hat{k}$, $2\hat{i}-\hat{j}+7\hat{k}$ and $7\hat{i}-\hat{j}+23\hat{k}$

    (ii) $\hat{i}+2\hat{j}+3\hat{k}$, $2\hat{i}+\hat{j}+3\hat{k}$ and $\hat{i}+\hat{j}+\hat{k}$
81. A vector $\overline{OP}$ is inelined to OX at 45° and OY at 60°. Find the angle at which $\overline{OP}$ is inclined to OZ.
82. A vector $\vec{r}$ has length 21 and direction ratios 2, –3, 6. Find the direction cosines and components of $\vec{r}$, makes an acute angle with x-axis.

83. Find the direction cosines of the vector joining the points A(1, 2, –3) and B(–1, –2, 1), directed from A to B.

EXERCISE-2

1. Can a vector have direction angle 45°, 60°, 120° ?
2. A vector makes an angle of $\frac{\pi}{4}$ with each of x-axis and y-axis. Find the angle made by it with the z-axis.
3. Find the direction cosines of the vector joining the points A(1, 2, –3) and B(–1, –2, 1), direction from A to B.
4. Show that the direction consines of a vector equally inclined to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.
5. If a unit vector $\vec{a}$ makes an angle $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an accute angle θ with $\hat{k}$, then find θ and hence, the components of $\vec{a}$.
6. If $\vec{a}, \vec{b}, \vec{c}$ represent the sides of a triangle taken in order, then write the value of $\vec{a}+\vec{b}+\vec{c}$.
7. If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices A, B and C respectively, of a triangle ABC, write the value of $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}$.
8. If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the points A, B and C respectively, write the value of $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{AC}$.
9. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices of a triangle, then write the position vector of its centroid.
10. Write the position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ externally in the ratio 1:4, where $\vec{a} = 2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{b} = -\hat{i}+\hat{j}+\hat{k}$.
11. Write the direction cosines of the vector $\vec{r} = 6\hat{i}-2\hat{j}+3\hat{k}$.
12. A unit vector $\vec{r}$ makes angles $\frac{\pi}{3}$ and $\frac{\pi}{2}$ with $\hat{j}$ and $\hat{k}$ respectively and an acute angle θ with $\hat{i}$. Find θ.
13. If $\vec{a} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b} = 2\hat{i}+4\hat{j}+9\hat{k}$, find a unit vector parallel to $\vec{a}+\vec{b}$.
14. Write a unit vector in the direction of $\vec{b} = 2\hat{i}+\hat{j}+2\hat{k}$.
15. Find the position vector of the mid-point of the line segment AB, where A is the point (3, 4, –2) and B is the point (1, 2, 4).
16. Find a vector in the direction of $\vec{a} = 2\hat{i}-\hat{j}+2\hat{k}$, which has magnitude of 6 units.
17. What is the cosine of the angle which the vector $\sqrt{2}\,\hat{i}+\hat{j}+\hat{k}$ makes with y-axis ?